Affleck-Dine baryogenesis in anomaly-mediated SUSY breaking

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Abstract. It has been known that in anomaly-mediated SUSY breaking model Affleck-Dine baryogenesis does not work due to trapping of Affleck-Dine field into charge-breaking minima. We show that when finite-temperature effect is properly taken into account and if reheating temperature is relatively high, the problem of falling into charge breaking global minima can be avoided and hence Affleck-Dine baryogenesis works. Moreover, for the LH_u flat direction we obtain a constraint on the mass of neutrino.

1. Introduction

The origin of baryon asymmetry, or matter-anti-matter asymmetry, is one of the most interesting topics for both cosmology and particle physics. From WMAP three year results [1],

$$\eta = \frac{n_B}{n_\gamma} \simeq (6.1 \pm 0.2) \times 10^{-10}. \tag{1}$$

is obtained. Primordial big-bang nucleosynthesis explains the observed light element abundances for about the same value of η [2]. To explain this value, various baryogenesis mechanism have been considered.

Supersymmetry (SUSY) [3] is the most attractive candidate as physics beyond the standard model. Thus it is worthwhile to construct the baryogenesis model based on SUSY. Affleck-Dine mechanism [4] is one of the most studied baryogenesis scenario in the framework of supersymmetric standard model [5]. This uses the dynamics of flat directions existing in the minimal supersymmetric standard model (MSSM), which is constructed of the scalar fields having flat potential in supersymmetric limit at renormalizable level. The angular motion of flat directions ϕ in complex plane generates lepton or baryon current,

$$n = iN(\dot{\phi}^* \phi - \phi^* \dot{\phi}) = N|\phi|^2 \dot{\theta} \tag{2}$$

where N is the constant determined by the flat direction, and we have defined $\phi = |\phi| e^{i\theta}$. How this angular motion is generated or what amount of baryon asymmetry can be created depends on the type of flat direction, temperature of the universe and the mechanism of supersymmetry breaking. Finite-temperature effects [7, 8] are another important correction, which can significantly affect the whole dynamics.

Furthermore, formation of Q-balls makes the usual Affleck-Dine scenario complicated [9, 10]. The property of Q-balls strongly depends on SUSY breaking mechanism. In gravity-mediation scenario, Q-ball is unstable against decay into nucleons and created baryon number is finally converted to ordinary matter [11, 12]. In gauge-mediation scenario, however, Q-ball is stable and only evaporation and diffusion effects can extract the baryon number from Q-balls [13, 14, 15]. On the other hand, Q-ball can explain naturally why the dark matter and baryon number density are roughly the same order in the universe in some specific models [16]. Therefore, when considering Affleck-Dine baryogenesis scenario, one must carefully trace the dynamics of flat directions taking into account various effects.

Anomaly-mediation models [17] have attractive feature on phenomenological point of view. In this scenario, SUSY breaking effect in hidden sector are transmitted to observable sector through super-Weyl anomaly. This predicts model-independent generic feature at low energy physics insensitive to physics at high energy scale. As a consequence, the flavor problem existing in usual gravity-mediation scenario is relaxed, and the new possibility of wino-like dark matter is provided [18, 19]. Gravitino mass becomes two orders of magnitude larger than that of gravity-mediation case, which also solves the gravitino problem. However, as explained in the next section, Affleck-Dine baryogenesis in anomaly-mediation models has been revealed to be difficult [20, 19].

In this paper, we study Affleck-Dine mechanism in anomaly-mediation models including finite-temperature effects, which was missed in previous literature. It is shown that when finite-temperature effect is properly taken into account Affleck-Dine baryogenesis works. We also discuss Q-ball formation and its consequences.

This paper is organized as follows. First in Sec. 2, potentials for flat directions including finite-temperature effects are given. In Sec. 3 we describe the dynamics of $n = 4$ flat directions when finite-temperature effects are taken into account. The case with $n = 6$ flat direction is discussed in sec. 4. In Sec. 5 the effects of Q-ball formation is described and we conclude in Sec. 6

2. Potential of Affleck-Dine field in anomaly-mediation model

2.1. Zero-temperature potential

First, we summarize the standard scenario for Affleck-Dine baryogenesis neglecting the finite-temperature effect. Potentials of flat directions in the MSSM are exactly flat in renormalizable and supersymmetric limit, but lifted by non-renormalizable terms and supersymmetry breaking effect. If we parameterize a flat direction ϕ, non-renormalizable superpotential

$$W = \frac{\phi^n}{nM^{n-3}} \tag{3}$$

generates the potential for ϕ,

$$V = \frac{|\phi|^{2(n-1)}}{M^{2n-6}} \tag{4}$$

where M denotes some cut-off scale ‡. All MSSM flat directions are known to be lifted up to $n = 9$ gauge-invariant superpotentials. Including supersymmetry breaking effects, the potential would be

$$V = (m_\phi^2 - cH^2)|\phi|^2 + \left\{ a_m \frac{m_{3/2}\phi^n}{nM^{n-3}} + a_H \frac{H\phi^n}{nM^{n-3}} + \text{h.c.} \right\} + \frac{|\phi|^{2(n-1)}}{M^{2n-6}}, \tag{5}$$

where H is Hubble parameter, c, a_m and a_H are constants of $O(1)$. Initially, the ϕ field is trapped at the minimum determined by the negative Hubble mass term $-H^2|\phi|^2$ and the highest term $|\phi|^{2(n-1)}/M^{2n-6}$ as

$$|\phi| \simeq (HM^{n-3})^{\frac{1}{n-2}}. \tag{6}$$

The ϕ field traces this minimum until H becomes less than m_ϕ and it begins to oscillate around the origin. The angular direction of ϕ is determined by the Hubble-induced A-term until this epoch. If $m_\phi \sim m_{3/2}$ as expected in gravity mediation, at the same time when ϕ begins to oscillate, the soft A-term begins to dominate over the Hubble-induced A-term, and this causes the kick in the angular direction. Finally the ϕ field shows the $U(1)$ conserving elliptical motion around their minimum. At this stage, the created baryon number is conserved. This is the standard scenario for creating baryon asymmetry in the Affleck-Dine mechanism.

In anomaly-mediation models, however, there is a subtlety. Since soft mass is loop-suppressed in anomaly-mediation, we expect $m_\phi \sim m_{3/2}/(8\pi^2)$. If we assume soft masses should be $100 \sim 1000$ GeV, the natural order of gravitino mass is estimated to be $10 \sim 100$ TeV, two orders of magnitude larger than that of gravity-mediation case. This is a good feature for avoiding gravitino problem. For such large gravitino mass, its lifetime naturally becomes shorter than 1 sec, which does not much affect BBN. In fact, even if its hadronic branching ratio is order one, there is no upper bound on the reheating temperature if gravitino mass is as large as 100TeV [21]. However, this invalidates the usual Affleck-Dine baryogenesis scenario because the potential of flat direction (5) has charge and/or color breaking global minima with

$$|\phi|_{\min} \sim \left(\frac{|a_m|}{n-1} m_{3/2} M^{n-3} \right)^{\frac{1}{n-2}}. \tag{7}$$

The minimum value of the potential becomes

$$V(|\phi|_{\min}) \sim -\frac{n-2}{n} M^4 \left(\frac{|a_m|}{n-1} \frac{m_{3/2}}{M} \right)^{\frac{2n-2}{n-2}}, \tag{8}$$

which is always negative for $n \geq 4$. This is not a problem if the relevant fields sit at the origin initially, since the decay rate of the false vacuum into a true charge breaking minimum is sufficiently small and the lifetime is longer than the age of the universe [20].

‡ We use the same symbol ϕ as a chiral superfield or its scalar part.

But in Affleck-Dine set-up, the corresponding flat direction should have large field value tracking their minimum (6), and finally fall into charge breaking minima (7). This is a fundamental problem when applying Affleck-Dine mechanism to anomaly-mediation models.

There is an attempt for Affleck-Dine baryogenesis in anomaly-mediation models based on gauged $U(1)_{B-L}$ symmetry [22]. This uses the fact that $U(1)_{B-L}$ breaking effect stops the ϕ field moving beyond the $U(1)_{B-L}$ breaking scale v due to D-term contribution. If v is smaller than the field value corresponding to the hill of the potential (5),

$$|\phi|_{\rm hill} \sim \left(\frac{m_\phi^2 M^{n-3}}{m_{3/2}}\right)^{\frac{1}{n-2}}, \tag{9}$$

we do not need to worry about falling into the unphysical global minima. This is an appealing feature, but this model relies on the hypothesis of gauged $U(1)_{B-L}$ symmetry. In the following, we show that finite-temperature effect enables us to obtain baryon asymmetry and avoid the charge breaking minima in anomaly-mediation models without any further assumption beyond MSSM.

2.2. Finite-temperature effect

Thermal effects modify the potential of flat directions. First, couplings of flat directions with other particles ψ yields thermal mass term [7] given by

$$\sum_{f_k|\phi|<T} c_k f_k^2 T^2 |\phi|^2, \tag{10}$$

where c_k is a constant of $O(1)$ and f_k denotes gauge or Yukawa coupling relevant for the flat direction. Note that when $f_k|\phi| > T$, ψ receives a large mass of order $f_k|\phi|$ and can not be thermalized, and hence ϕ does not feel thermal mass.

It was also pointed out that the following form of the potential [8]

$$V \sim a\alpha(T)^2 T^4 \log\left(\frac{|\phi|^2}{T^2}\right) \tag{11}$$

should be included for the potential of flat directions, where a is a order 1 constant determined by two-loop finite temperature effective potential. For LH_u direction, $a = 9/8$. When this term dominates, it is possible that flat direction begins to oscillate due to thermal logarithmic term. The epoch of onset of oscillation is determined by

$$H_{\rm OS}^2 \sim m_\phi^2 + \sum_{f_k|\phi|<T} c_k f_k^2 T^2 + a\alpha(T)^2 \frac{T^4}{|\phi|^2}. \tag{12}$$

Details of the dynamics depend on corresponding flat directions and somewhat complicated [23, 24]. Now let us investigate it for $n = 4$ and $n = 6$ case.

3. The case of $n = 4$ flat direction

In this section, we describe the dynamics of AD field for $n = 4$ based on the potential given in the previous section, concentrating on LH_u direction particularly. This is because Affleck-Dine baryogenesis can not work successfully for other $n = 4$ directions (see Sec.3.3).

3.1. Dynamics of $n = 4$ flat direction

From eq.(12) for $n = 4$, early oscillation due to the thermal log term occurs when

$$T_R \gtrsim \frac{m_\phi}{\alpha(T)} \left(\frac{M}{M_P}\right)^{\frac{1}{2}}, \tag{13}$$

where T_R is reheating temperature after inflation, and in this case the Hubble parameter at start of the oscillation is

$$H_{\rm OS} \sim \alpha T_R \left(\frac{M_P}{M}\right)^{\frac{1}{2}}. \tag{14}$$

Here we have used $T \sim (T_R^2 H M_P)^{1/4}$ and $|\phi| \sim (HM)^{1/2}$. For natural range of cut-off scale M, early oscillation naturally takes place unless reheating temperature T_R is unnaturally low.

It should be noticed that, for sufficiently high reheating temperature, this thermal logarithmic potential can hide the unwanted valley of the potential. Substituting $n = 4$ into eq.(8), the minimum of the zero-temperature potential is given by

$$V(|\phi|_{\rm min}) \sim -\frac{1}{54} m_{3/2}^3 M. \tag{15}$$

In order to avoid falling into this minimum, at least $\alpha^2 T^4 \gtrsim |V(|\phi|_{\rm min})|$ is required at the beginning of oscillation. This leads to

$$T_R \gtrsim \alpha^{-1} m_{3/2} \left(\frac{M}{M_P}\right)^{\frac{1}{2}}. \tag{16}$$

If we assume $M \sim M_P$, $T_R \gtrsim 10 m_{3/2} \sim 10^{5-6}$ GeV is needed to satisfy the above condition. Note that thermal mass term can not dominate over the thermal logarithmic term at this epoch. We have checked numerically that the above condition is almost sufficient to drive the Affleck-Dine field into the origin §.

Thus, for high enough reheating temperature Affleck-Dine baryogenesis can work irrespective of the charge-breaking minima of the potential. To confirm this statement,

§ As an another condition for avoiding global minima, one may require $|\phi| < |\phi|_{\rm hill}$ at $H \lesssim m_{3/2}$, which is the epoch soft A-term begins to dominate over the Hubble-induced A-term. This is achieved when the following condition is satisfied,

$$T_R \gtrsim \alpha^{-1} \frac{m_{3/2}^2}{m_\phi} \left(\frac{M}{M_P}\right)^{\frac{1}{2}}. \tag{17}$$

But in fact this condition is too strong. This condition is sufficient, but not always necessary. Numerical calculation shows the condition (16) is almost sufficient.

we have performed numerical calculation with full scalar potential including finite-temperature effect. For simplicity, we set $M = M_P$ (in next section, we see that this choice is valid to obtain a proper amount of baryon asymmetry). As explained above, $T_R \gtrsim 10^6$GeV is needed to obtain appropriate motion of the flat direction. In Fig. 1, we show the result when $T_R = 10^6$GeV and $m_{3/2} = 100$TeV. Clearly one can see ϕ field falls into the origin with angular motion, which indicates that the Affleck-Dine baryogenesis works. The resultant baryon asymmetry is analyzed in the next section.

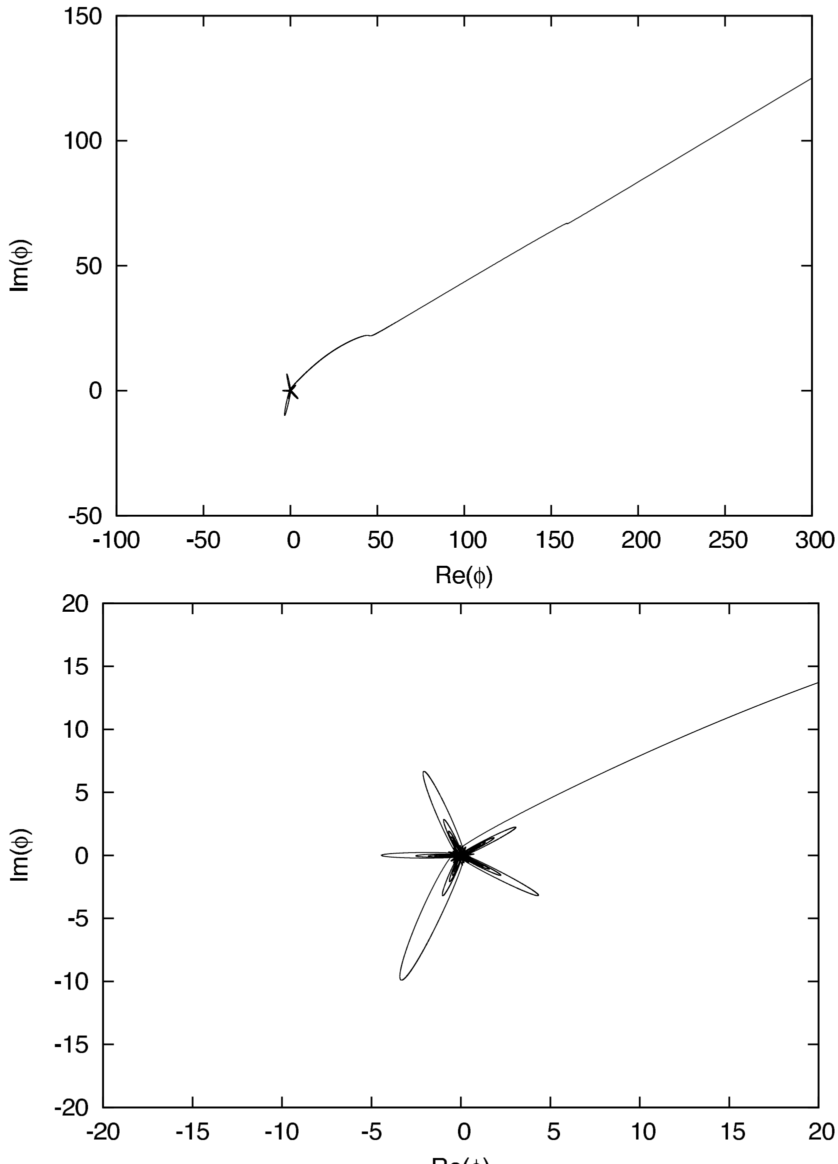

Figure 1. (Upper) Typical motion of the field ϕ in the complex plane when $T_R = 10^6$GeV. Field value is normalized by 10^{10}GeV. (Lower) The motion near the origin is magnified

On the other hand, in Fig. 2 the result for $T_R = 10^5$GeV is shown. In this case, finite-temperature effect is insufficient to take the ϕ field into the origin, and finally it is trapped at the charge-breaking displaced minimum (7). Therefore, in order to make Affleck-Dine scenario successful in anomaly-mediation models, at least reheating temperature $T_R \gtrsim 10^6$GeV is needed though this value varies depending on the cut-off scale M.

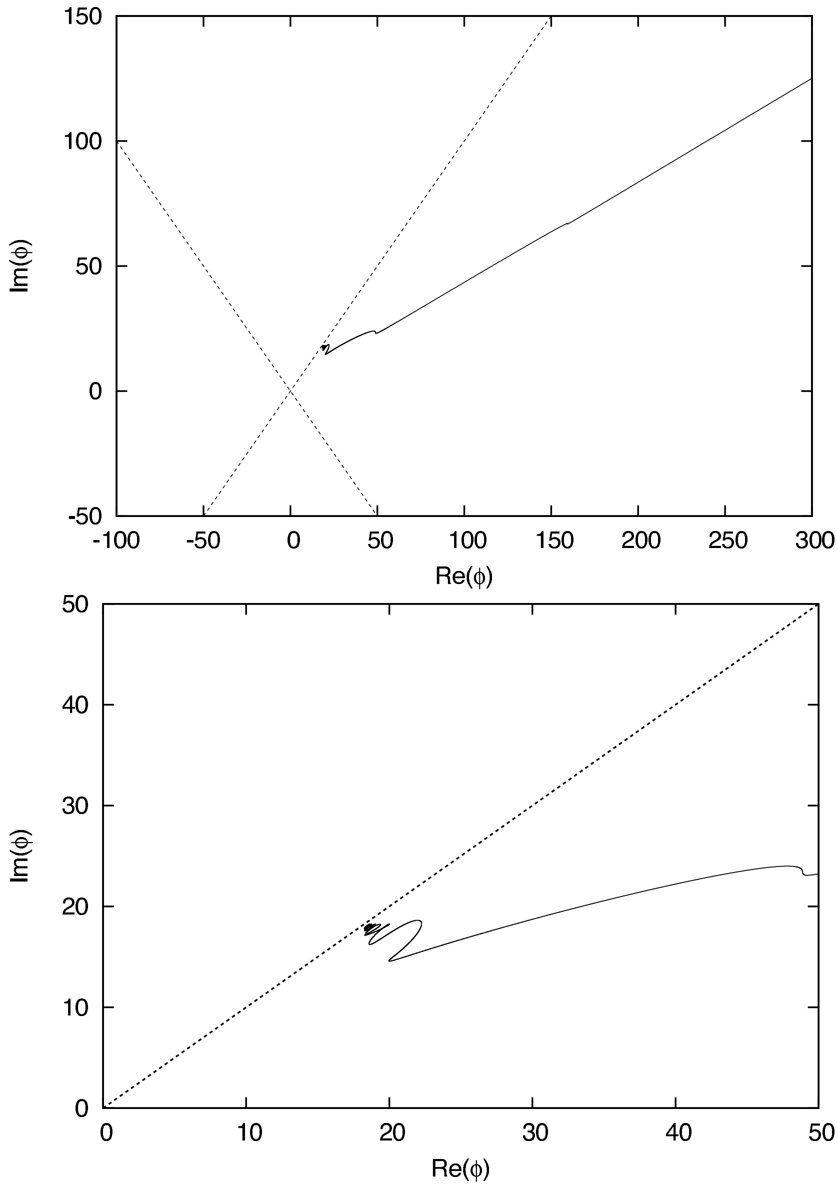

Figure 2. (Upper) Typical motion of the field ϕ in the complex plane when $T_R = 10^5$GeV. Dotted line represents the valley of the potential from soft A-term. (Lower) The motion near the minimum is magnified

3.2. Baryon number generation

Let us estimate the baryon number created in this process. Actually the LH_u direction generates lepton number, but electroweak sphaleron process quickly converts it into baryon number [25, 26]. From eq.(2) and equation of motion of ϕ

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi^*} = 0, \tag{18}$$

we obtain

$$\dot{n} + 3Hn = 2N\mathrm{Im}\left(\phi\frac{\partial V}{\partial\phi}\right). \tag{19}$$

Baryon number per comoving volume is almost fixed at the beginning of oscillation. Integrating above expression, we obtain

$$n(t_{\mathrm{OS}}) \;\sim\; N\delta_e m_{3/2}(H_{\mathrm{OS}}M^{n-3})^{\frac{2}{n-2}}, \tag{20}$$

where $\delta_e = \sin(\arg a_m + n\arg\phi)$ represents the degree of CP violation, which is naturally expected to be of order 1. Notice that the reheating in which the inflaton decays completely takes place after the AD field starts oscillation. After reheating the baryon-to-entropy ratio is estimated as

$$\frac{n}{s} = \frac{n(t_R)}{s(t_R)} \sim N\frac{\delta_e m_{3/2}T_R}{H_{\mathrm{OS}}^2 M_P^2}(H_{\mathrm{OS}}M^{n-3})^{\frac{2}{n-2}}. \tag{21}$$

In the present LH_u case, H_{OS} is given by eq. (14),

$$\frac{n}{s} \sim 10^{-10}\delta_e\left(\frac{0.1}{\alpha}\right)\left(\frac{m_{3/2}}{50\mathrm{TeV}}\right)\left(\frac{10^{-7}\mathrm{eV}}{m_\nu}\right)^{\frac{3}{2}} \tag{22}$$

where we have used the fact that m_ν is given by

$$m_\nu \sim \frac{\langle H_u\rangle^2}{M} = \frac{\sin^2\beta}{M}(174\mathrm{GeV})^2 \tag{23}$$

with $\sin\beta \sim 1$. Interestingly, the baryon-to-entropy ratio (22) is independent of reheating temperature [24]. This is because lowering reheating temperature tends to generate small baryon-to-entropy ratio, but on the other hand, the epoch of oscillation due to the thermal logarithmic term becomes late and leads to larger baryon number. As a result, these two effects cancel and baryon-to-entropy ratio becomes independent of reheating temperature, as far as the only requirement $T_R \gtrsim 10^5$GeV is satisfied. The mass of neutrino should be less than $\sim 10^{-7}$ eV for successful baryogenesis. Note that due to largeness of $m_{3/2}$, constraint on the neutrino mass for successful baryogenesis is weaker than that of usual gravity-mediation case [24].

3.3. Some comments

Some comments are in order.

First, we comment on instability of AD condensate and Q-ball formation in our scenario. If Q-balls are formed, almost all charges are trapped into them [12] and the subsequent evolution becomes complicated. When finite-temperature effect is neglected, whether the relevant AD condensate is stable or not is determined by quantum correction to the mass squared

$$m^2\left\{1 + K\log\left(\frac{|\phi|^2}{M^2}\right)\right\}. \tag{24}$$

If $K < 0$, instability develops and finally Q-balls are formed. For LH_u direction, K is positive and Q-balls are not formed [27]. But when oscillation of AD condensates occurs due to finite-temperature effect, instability can develop and result in formation of Q-balls. However, the resultant charge of Q-balls are so small that they can not survive until temperature becomes lower than the electroweak scale as shown in Sec. 5. Thus, Q-ball formation does not have any non-trivial cosmological consequence, and we do not bother to worry about complication due to Q-ball formation.

Then, what about $n = 4$ flat directions other than LH_u? It is known that other $n = 4$ directions in MSSM conserve $B - L$ [28]. Thus sphaleron effects completely wash out the created baryon asymmetry. Although sufficiently large Q-ball can protect baryon asymmetry from sphaleron effect, such large Q-balls do not seem to be created in the presence of early oscillation as described above.

Finally, we comment on the dark matter candidate in our scenario. Thermal relic of wino LSP in anomaly-mediation models can not explain the observed dark matter density due to their large annihilation cross section [29], unless it is as heavy as 2 TeV. Non-thermally produced wino from Q-ball decay is possible candidate [19], but in our model large Q-balls are not formed, so this possibility is excluded. Thus, we can not explain in this model both the baryon asymmetry and dark matter simultaneously, and we need other particle such as axion to account the present density of matter in the universe.

4. The case of $n = 6$ flat direction

The similar analysis can be applied to $n = 6$ flat direction. There are some flat directions in MSSM which are lifted by $n = 6$ non-renormalizable superpotentials. The most interesting direction is udd direction, which is responsible for B-ball baryogenesis [11]. In usual gravity-mediation case, Q-balls associated with AD condensate corresponding to $n = 6$ flat direction can survive below the freeze-out temperature of LSP, and subsequent decay produce baryon number and non-thermal LSP. In some models, this can naturally explain both the baryon asymmetry and dark matter [16, 19].

However, in our model, to avoid charge or color breaking minima, early oscillation due to the thermal logarithmic term is needed. Since the udd direction is expected to have a negative coefficient for thermal logarithmic term ($a < 0$ in eq. (11)), early oscillation unlikely occurs. Thus, the argument similar to LH_u case can not be applied to this direction. But for other $n = 6$ direction, e.g. LLe direction, it may be possible that AD baryogenesis in anomaly-mediation models works. In this section, we investigate this possibility.

4.1. Dynamics of $n = 6$ flat direction

First, we study the condition for early oscillation to occur. Using $|\phi| \sim (HM^3)^{1/4}$, we obtain

$$T_R \gtrsim \frac{1}{\alpha}\left(\frac{m_\phi^3 M^3}{M_P^2}\right)^{\frac{1}{4}} \sim 3\times 10^5 \mathrm{GeV}\left(\frac{m_\phi}{100\mathrm{GeV}}\right)^{\frac{3}{4}}\left(\frac{M}{10^{15}\mathrm{GeV}}\right)^{\frac{3}{4}}. \qquad (25)$$

Compared with $n = 4$ flat direction, higher reheating temperature or smaller cut-off scale are needed.

Next, in order to avoid the unphysical minima, the thermal log must hide the valley of the zero-temperature potential. From eq.(8) with $n = 6$,

$$V(|\phi|_{\min}) \simeq -\frac{2}{75\sqrt{5}} m_{3/2}^{\frac{5}{2}} M^{\frac{3}{2}}. \qquad (26)$$

We require $\alpha^2 T^4 \gtrsim |V(|\phi|_{\min})|$ at the beginning of oscillation. This leads to

$$\begin{aligned} T_R &\gtrsim \alpha^{-1} M^{\frac{3}{4}} M_P^{-\frac{1}{2}} m_{3/2}^{\frac{3}{4}} \\ &\sim 8\times 10^5 \mathrm{GeV}\left(\frac{0.1}{\alpha}\right)\left(\frac{M}{10^{15}\mathrm{GeV}}\right)^{\frac{3}{4}}\left(\frac{m_{3/2}}{10\mathrm{TeV}}\right)^{\frac{3}{4}}. \qquad (27) \end{aligned}$$

Thus, $T_R \gtrsim 10^6$ GeV is necessary for $M \sim 10^{15}$GeV and $m_{3/2} \sim 10$TeV. When this constraint on the reheating temperature is satisfied, the dynamics of flat direction is similar to that of LH_u direction studied in the previous section. Until H becomes lower than H_{OS}, the radial component of AD field is trapped by the instant minimum determined by the negative Hubble mass term and the non-renormalizable term. Then, AD field begins to oscillate around their origin, and receives angular kick. The baryon-to-entropy ratio is almost fixed at this epoch. Although this reheating temperature seems rather high, gravitino mass is heavy enough to decay before the BBN epoch in anomaly-mediation models. So high reheating temperature is not a problem. We have checked numerically that for $T_R \gtrsim 10^6$GeV, ϕ rolls down to the origin without trapped by the displaced minima.

4.2. Baryon number generation

If the constraint on the reheating temperature (27) is satisfied, Affleck-Dine mechanism can work with no more difficulty. From eq.(21),

$$\frac{n}{s} \sim \frac{1}{9}\delta_e m_{3/2} T_R M_P^{-2}\left(\frac{M}{H_O}\right)^{\frac{3}{2}} \qquad (28)$$

where H_{OS} is given by

$$H_{\mathrm{OS}} \sim (a\alpha^2 T_R^2 M_P M^{-\frac{3}{2}})^{\frac{2}{3}} \qquad (29)$$

from eq. (12). Substituting this into eq. (28), we obtain

$$\begin{aligned} \frac{n}{s} &\sim \frac{\delta_e m_{3/2} M^3}{9\alpha^2 T_R M_P^3} \qquad (30) \\ &\sim 10^{-10}\delta_e\left(\frac{0.1}{\alpha}\right)^2\left(\frac{m_{3/2}}{10\mathrm{TeV}}\right)\left(\frac{10^8\mathrm{GeV}}{T_R}\right)\left(\frac{M}{10^{16}\mathrm{GeV}}\right)^3. \qquad (31) \end{aligned}$$

Thus, we can obtain a proper amount of baryon asymmetry with parameters consistent with the constraint (27).

5. Q-ball formation

In the previous section, we have showed that the Affleck-Dine mechanism can create a sizable baryon asymmetry in anomaly-mediation models with rather high reheating temperature. But it is non-trivial matter whether the created baryon asymmetry actually provides the baryon density of the universe which is required by BBN or CMB anisotropy. This is because Q-balls may be formed and almost all baryon charge are trapped into them. If charge of Q-balls is large and they are stable, the evolution of the AD field and resultant baryon asymmetry of the universe may be changed significantly. In this section, we see that Q-ball formation in our models have no great importance on cosmology.

Generally, perturbations to the AD fields ϕ grow when their potential is less steep than ϕ^2 due to the negative pressure. In our models, oscillation of the AD fields is controlled by thermal logarithmic term, and hence the instability develops into formation of Q-balls. Since this is similar to gauge-mediation type Q-ball, the same analysis as gauge-mediation case can be applied.

First note that although the whole dynamics of Q-ball formation is highly non-linear, the radius of Q-balls is determined by the fastest growing mode in perturbative analysis. This is checked by numerical calculation based on lattice simulation [12]. Roughly speaking, the wave length of the most growing mode is comparable to the Hubble radius at the epoch of oscillation of AD fields,

$$\frac{|\mathbf{k}|}{a} \sim H_{\rm OS} \sim \frac{T_{\rm OS}^2}{|\phi(t_{\rm OS})|}, \tag{32}$$

where $T_{\rm OS}$ denotes the temperature at the beginning of oscillation and is given by $T_{\rm OS} \sim (T_R^2 H_{\rm OS} M_P)^{1/4}$. Thus, we expect early oscillation due to thermal effect tends to yield smaller Q-balls. The resultant charge inside Q-balls Q is given by $Q \sim H_{\rm OS}^{-3} n_B(t_{\rm OS})$. Here we note that eq.(20) can be written in the convenient form $n_B(t_{\rm OS}) \sim \epsilon H_{\rm OS}|\phi(t_{\rm OS})|^2$, where ϵ denotes ellipticity of the orbit of AD field, $\epsilon = m_{3/2}/H_{\rm OS}$. The result is,

$$Q \sim \epsilon \frac{|\phi(t_{\rm OS})|^4}{T_{\rm OS}^4} \sim \epsilon \left(\frac{M}{T_{\rm OS}}\right)^{4\frac{n-3}{n-1}}. \tag{33}$$

In fact, Q-ball formation begins slightly later than the oscillation of AD field and the number of Q-balls per Hubble horizon is expected to be more than one. In ref. [30], it is found that the maximum charge of Q-balls is fitted by the formula when the ellipticity ϵ is much smaller than 1,

$$Q_{\rm max} = \beta \left(\frac{\phi(t_{\rm OS})}{T_{\rm OS}}\right)^4 \tag{34}$$

with $\beta \sim 6\times10^{-4}$ from lattice simulation. This agrees with eq. (33) except for numerical factor which can not be determined analytically. Eq. (34) is independent of ϵ because anti-Q-balls are also produced so that the net baryon number is small. In the case of early oscillation, since it is expected that $\epsilon \ll 1$, we use eq. (34) in the following. Now we estimate the charges of Q-balls in $n=4$ and $n=6$ case, respectively.

5.1. $n = 4$ case

From eq. (14), the temperature at the oscillation $T_{\rm OS}$ is given by

$$T_{\rm OS} \sim (\alpha T_R^3 M_P^{\frac{3}{2}} M^{-\frac{1}{2}})^{\frac{1}{4}}. \tag{35}$$

Thus, the charge of Q-balls is estimated as

$$\begin{aligned} Q &\sim \beta \left(\frac{M}{T_{\rm OS}}\right)^{\frac{4}{3}} \\ &\sim 3\times10^9 \left(\frac{0.1}{\alpha}\right)^{\frac{1}{3}} \left(\frac{\beta}{6\times10^{-4}}\right) \left(\frac{10^6{\rm GeV}}{T_R}\right) \left(\frac{M}{M_P}\right)^{\frac{3}{2}}. \end{aligned} \tag{36}$$

With the aid of eq. (21), this can be rewritten as

$$\begin{aligned} Q &\sim 1\times10^9\ \delta_e^{-1} \left(\frac{\alpha}{0.1}\right)^{\frac{2}{3}} \left(\frac{\beta}{6\times10^{-4}}\right) \\ &\quad\times \left(\frac{10^6{\rm GeV}}{T_R}\right) \left(\frac{50{\rm TeV}}{m_{3/2}}\right) \left(\frac{n_B/s}{10^{-10}}\right). \end{aligned} \tag{37}$$

As shown in Appendix A, in order to survive evaporation in high temperature plasma, $Q \gtrsim 10^{18}$ is needed. Thus, even if Q-balls are formed, they are expected to evaporate completely well above $T \sim 100{\rm GeV}$ and the estimation of baryon asymmetry in Sec. 3 need not be changed.

5.2. $n = 6$ case

In this case, $H_{\rm OS}$ is given by eq. (29). Then, the temperature at the oscillation is

$$T_{\rm OS} \sim \alpha^{\frac{1}{3}} T_R^{\frac{5}{6}} M^{-\frac{1}{4}} M_P^{\frac{5}{12}}. \tag{38}$$

The charge of Q-balls in the $n=6$ case becomes

$$\begin{aligned} Q &\sim \beta \left(\frac{M}{T_{\rm OS}}\right)^{\frac{12}{5}} \\ &\sim 2\times10^{10} \left(\frac{0.1}{\alpha}\right)^{\frac{4}{5}} \left(\frac{\beta}{6\times10^{-4}}\right) \left(\frac{10^7{\rm GeV}}{T_R}\right)^2 \left(\frac{M}{10^{15}{\rm GeV}}\right)^3. \end{aligned} \tag{39}$$

Using eq. (31), this is rewritten in terms of n/s as

$$\begin{aligned} Q &\sim 2\times10^{12}\ \delta_e^{-1} \left(\frac{\alpha}{0.1}\right)^{\frac{6}{5}} \left(\frac{\beta}{6\times10^{-4}}\right) \\ &\quad\times \left(\frac{10^7{\rm GeV}}{T_R}\right) \left(\frac{10{\rm TeV}}{m_{3/2}}\right) \left(\frac{n_B/s}{10^{-10}}\right). \end{aligned} \tag{40}$$

This is also so small that Q-balls can not survive evaporation.

6. Conclusion

We have investigated the Affleck-Dine mechanism in anomaly-mediated SUSY breaking models. In contrast to previous studies, we have found that early oscillation due to finite-temperature effects can drive flat directions into the correct vacuum and a proper amount of baryon asymmetry can be generated for neutrino mass about $m_\nu \lesssim 10^{-7}$ eV in the case of LH_u direction. Our scenario requires somewhat high reheating temperature, but this leads to no cosmological difficulties such as gravitino problem since gravitino is heavy enough and decay before the onset of BBN in anomaly-mediation models.

We have also investigated the same mechanism for $n = 6$ flat direction case. It is found that for natural range of parameters, proper amount of baryon asymmetry can be obtained. Furthermore, we also have discussed consequences of Q-ball formation. It is shown that all Q-balls evaporate in high-temperature plasma. Therefore, the Q-ball formation does not complicate the baryogenesis process.

Appendix A. Evaporation of Q-balls

A Q-ball is a non-topological soliton whose stability is ensured by global $U(1)$ symmetry. But it can release its charge in some manner. Here we concentrate on the following two process. The first is decay of AD field into pair of fermions or lighter bosons, and the second is evaporation and diffusion effects in thermal bath. In this Appendix, we give a rough estimation of the total amount of evaporated charge from Q-balls.

Appendix A.1. Decay into light particles

It is known that the energy per charge inside Q-balls of gauge-mediation type is proportional to $Q^{-1/4}$ [13]. Thus, for large enough Q, Q-balls are stable against decay into nucleons. The energy per unit charge of gauge-mediated type Q-ball is given by

$$\frac{E_Q}{Q} \sim TQ^{-\frac{1}{4}}. \tag{A.1}$$

This leads to the stability condition,

$$Q > \left(\frac{T}{m_N}\right)^4 \tag{A.2}$$

where m_N denotes the mass of nucleon, $m_N \sim 1$GeV. Thus, for $T \lesssim Q^{1/4}$GeV, Q-balls become stable against decay into nucleons, although decay into light neutrinos is possible for leptonic Q-balls (L-balls). But as temperature becomes low, as explained in the next subsection, gauge-mediated type Q-balls dominated by thermal logarithmic potential are deformed or converted into gravity-mediated type. Even if Q-balls survive from evaporation, eventually they decay into free fermions or bosons since gravity-mediated type Q-balls have energy-to-charge ratio comparable to m_ϕ, which is much larger than m_N.

Thus, we consider the decay process of Q-balls into fermion pair. This occurs only from the surface of Q-ball since Pauli exclusion principle forbids the decay into fermions inside Q-ball [31]. This sets upper bound on the decay rate of Q-balls which can easily be saturated,

$$\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)_{\text{fermion}} \leq \frac{A\omega^3}{192\pi^2}, \tag{A.3}$$

where A denotes the surface area of Q-balls. Decay into pair of bosons is possible and may be largely enhanced compared with the case of fermions if there exists lighter scalar fields than AD fields. However, since this bosons become heavy inside Q-balls, the decay into lighter bosons only occurs through loop diagrams suppressed by the large effective mass [11], which leads to the enhancement factor $f_s \lesssim 10^3$, defined by

$$\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)_{\text{boson}} = f_s \left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)_{\text{fermion}}. \tag{A.4}$$

Using $A \sim 4\pi R_Q^2 \sim 4\pi |K|^{-1} m_\phi^{-2}$ for gravity-mediated type Q-balls, we obtain the decay temperature of Q-balls

$$T_d \sim 18\text{GeV}\sqrt{f_s}\left(\frac{0.01}{|K|}\right)^{\frac{1}{2}}\left(\frac{m_\phi}{100\text{GeV}}\right)^{\frac{1}{2}}\left(\frac{10^{18}}{Q}\right)^{\frac{1}{2}}. \tag{A.5}$$

For sufficiently large Q, this can become lower than the freeze-out temperature of dark matter, $T_f \sim m_{\mathrm{DM}}/20$. If this is the case, wino dark matter, which is the natural consequence of anomaly mediation model, produced by Q-ball decay can amount to desired abundance of dark matter [19]. But it should be noticed that *udd* direction is invalid because the coefficient of thermal logarithmic term is expected to be negative and hence the early oscillation does not occur. Furthermore, if one wants to explain baryon asymmetry and dark matter in this scenario, *LLe* or other pure leptonic direction does not work either, since created lepton number is protected from sphaleron effect inside the Q-ball until they decay at temperature below the electroweak scale. Other flat directions lifted by $n = 6$ superpotential is attractive candidates [28], but to obtain large Q is difficult in our scenario (see eqs.(37) and (40)).

Appendix A.2. Evaporation and diffusion

Q-ball formation is non-adiabatic process, and almost all charges are trapped into the Q-balls. This configuration is energetically stable, but in finite temperature environment, this is not always the case. In thermal bath there can exist free particles carrying charge surrounding Q-balls. The minimum of free energy is achieved when all charges are distributed in the form of Q-plasma. However, in actual situation, the evaporation of charge from Q-balls are not so sufficient in cosmic time scale. Thus, the mixture of plasma state and Q-ball state is realized. Then, it is important to know that how and what amount of charge of Q-balls is released into outer region.

Q-balls emit their charge through two process, evapolation [14] and diffusion [15]. Generally, as we see below, at high temperature the latter determines the emission rate of charge from Q-balls. First we estimate the evaporation rate from Q-balls. This occurs when the value of chemical potential of Q-balls (μ_Q) and surrounding plasma (μ_p) differs significantly. The evaporation rate is

$$\Gamma_{\rm evap} = -4\pi R_Q^2 \xi(\mu_Q - \mu_p) T^2, \tag{A.6}$$

where ξ is given by

$$\xi = \begin{cases} 1 & (T > m_\phi) \\ \left(\frac{T}{m_\phi}\right)^2 & (T < m_\phi). \end{cases} \tag{A.7}$$

But in fact around the edge of Q-balls, chemical equilibrium between plasma and Q-matter are achieved and charge inside the Q ball cannot come out at high temperature. Therefore, the charge in the 'atmosphere' of the Q ball should be taken away by diffusion in order for further charge evaporation. In this situation, charge transfer from inside Q-balls into plasma are determined by diffusion effect rather than above evaporation rate,

$$\Gamma_{\rm diff} = -4\pi D R_Q \mu_Q T^2 \sim -4\pi a T. \tag{A.8}$$

where we have used $D = a/T$ with $a = 4-6$, $\mu_Q \sim TQ^{-1/4}$ and $R_Q \sim T^{-1}Q^{1/4}$. when $T > m_\phi$, $\Gamma_{\rm diff} < \Gamma_{\rm evap}$ and the charge transfer is controlled by diffusion effects.

In our model, the thermal logarithmic potential dominates when AD field oscillates and Q-ball formation takes place. However, as temperature becomes low, the thermal effect ceases to be the dominant component of of the potential. As a rough estimation, this occurs when $T^4 \sim m_\phi^2|\phi_{\rm hill}|^2$, that is, $T \sim T_c \sim 10^6$GeV. If $|\phi| \ll |\phi_{\rm hill}|$ at this epoch, soft mass term determines the properties of Q-balls. If K in (24) is negative, the Q-ball configuration is a gravity-mediation type, $R_Q \sim m_\phi |K|^{-1/2}, \mu_Q \sim m_\phi$. Otherwise, Q-ball configuration does not stable any more and will collapse. In this case, Q-ball formation is irrelevant to baryogenesis. In the following, we consider the possibility that at $T < T_c$, the configuration of Q-balls is changed into gravity-mediation type. In our model, the reheating temperature T_R must be rather high as shown in previous sections. Thus, in the following, we assume $T_{\rm OS} > T_R > T_c > m_\phi$. Then, we obtain

$$\frac{\mathrm{d}Q}{\mathrm{d}T} = \begin{cases} \frac{32\pi a T_R^2 M_P}{3T^4} & (T > T_R) \\ \frac{16\pi a M_P}{T^2} & (T_R > T > T_c) \\ \frac{16\pi a M_P}{|K|^{1/2}T^2} & (T_c > T > T_*) \\ \frac{16\pi M_P}{|K| m_\phi T} & (T_* > T > m_\phi) \\ \frac{16\pi M_P T}{|K| m_\phi^3} & (T < m_\phi), \end{cases} \tag{A.9}$$

where T_* is defined by $T_* = a|K|^{1/2}m_\phi$. Although we have assumed $T_* > m_\phi$, this assumption does not much affect the following analysis. Now let us estimate the total amount of the evaporated charge ΔQ and examine whether or not Q-balls can survive in our model. Integrating above evaporation or diffusion rate, we obtain

$$\Delta Q(T > T_R) \qquad \sim \frac{32\pi a M_P}{9T_R}, \tag{A.10}$$

$$\Delta Q(T_R > T > T_c) \sim \frac{16\pi a M_P}{T_c}, \tag{A.11}$$

$$\Delta Q(T_c > T > T_*) \sim \frac{16\pi a M_P |K|^{-1/2}}{T_*}, \tag{A.12}$$

$$\Delta Q(T_* > T > m_\phi) \sim \frac{16\pi M_P |K|^{-1}}{m_\phi}, \tag{A.13}$$

$$\Delta Q(T < m_\phi) \sim \frac{8\pi M_P |K|^{-1}}{m_\phi}, \tag{A.14}$$

which are estimated as

$$\Delta Q(T > T_R) \sim 1 \times 10^{13} \left(\frac{10^7 \mathrm{GeV}}{T_R}\right), \tag{A.15}$$

$$\Delta Q(T_R > T > T_c) \sim 2 \times 10^{14} \left(\frac{10^6 \mathrm{GeV}}{T_c}\right), \tag{A.16}$$

$$\Delta Q(T_c > T > T_*) \sim 2 \times 10^{17} |K|^{-\frac{1}{2}} \left(\frac{10^3 \mathrm{GeV}}{T_*}\right), \tag{A.17}$$

$$\Delta Q(T_* > T > m_\phi) \sim 2 \times 10^{17} |K|^{-1} \left(\frac{10^3 \mathrm{GeV}}{m_\phi}\right), \tag{A.18}$$

$$\Delta Q(T < m_\phi) \sim 1 \times 10^{17} |K|^{-1} \left(\frac{10^3 \mathrm{GeV}}{m_\phi}\right). \tag{A.19}$$

From these, in order for Q-balls to survive evaporation,

$$Q \gtrsim 10^{18} \tag{A.20}$$

is required. Therefore, the charge transfer is enough to evaporate all charge from Q-balls for both $n = 4$ and $n = 6$ cases. Even if Q-balls are stable against decay into lighter particles, diffusion and evaporation effects can sufficiently transfer their charge into outer environment. Since our model requires high reheating temperature, this effect is unavoidable. After all, Q-balls can not survive until temperature becomes lower than about 100 GeV, where electroweak phase transition occurs.

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